Combinatorial Interpretations of Lucasnomials

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In high school, we learned about binomial coefficients $\binom{n}{k}$ and how they count the number of lattice paths from the origin to the point $(k, n-k)$. The calculation of these coefficients involves factorials, which is the multiplication of the current natural number with all the natural numbers that came before it (aside from 0 of course; otherwise this would be quite a boring concept). We can reframe this concept in terms of sequences by multiplying the current term with all the terms that came before it in the sequence. In this way we can extend the concept of binomial coefficients to something more general. Specifically, we explore Lucasnomials, the Lucas analog of binomial coefficients, which stem from the Lucas sequence: for variables s and t we define the nth term of the sequence as

$$\{n\}_{s,t} = s\,\{n-1\}_{s,t} + t\,\{n-2\}_{s,t}\,, \qquad \{0\}_{s,t} = 0,\ \{1\}_{s,t} = 1.$$

Note (1) that the Fibonacci numbers are a special type of Lucas sequence (where $s = t = 1$) and (2) that the binomial coefficients are special types of Lucasnomials (simply choose $s = 2$ and $t = -1$). It turns out that just like the binomial coefficients, these Lucasnomials are always positive integers! Thus, it is natural to ask if there is a combinatorial interpretation of Lucasnomials (i.e. an analog to the fact that binomial coefficients count lattice paths). In this talk we will explore this question by first looking at combinatorial interpretations of Fibonomial coefficients and then extending to the general case.

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7 p.m. in Park 245 or via Zoom

(https://brynmawr-edu.zoom.us/j/95807982212?pwd=aXBBMnFZMUUyWDQ1S1d3TGozc0t5Zz09)