

# Philadelphia Area Number Theory Seminar

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## Vinberg's Algorithm and the Classification of Maximal Hyperbolic Arithmetic Reflection Groups

**Abstract:**

Let  $Q$  be a quadratic form over  $\mathbb{Z}$  of signature  $(n, 1)$ . The orthogonal group  $O_Q^+(\mathbb{Z})$  is a discrete and finite-covolume subgroup of  $O_Q^+(\mathbb{R})$ , which can be viewed as the group of isometries of hyperbolic  $n$ -space. In many nice examples,  $O_Q^+(\mathbb{Z})$  is a reflection group, or has a finite-index normal reflection subgroup  $\Gamma$ . I will discuss Vinberg's algorithm, which finds the maximal reflection subgroup of  $O_Q^+(\mathbb{Z})$ . I will give computational examples and applications of this algorithm. Finally, I will discuss the state of current knowledge on the problem of classifying maximal hyperbolic arithmetic reflection groups: which quadratic forms  $Q$  yield a finite-index normal reflection subgroup  $\Gamma \subseteq O_Q^+(\mathbb{Z})$ , and which hyperbolic reflection groups can be obtained in this way? This talk will be expository, and will not contain new results.

**Thursday, March 3, 2022**  
**3:25 – 4:45 PM**

Swarthmore College  
Department of Mathematics and Statistics  
Science Center **149**

Informal refreshments at 3:10PM