

BI-CO MATHEMATICS COLLOQUIUM

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“Derivative Gain and Singularities for Degenerate Laplace Equation”

Monday, October 31, 2022

Talk at 4:00 – Park 338

Tea at 3:30 – Park 361, Math Lounge

Abstract:

Laplace's equation is ubiquitous both in undergraduate PDE textbooks and as a unifying equation of mathematical physics and beyond. Its solutions, called harmonic functions, in many ways resemble holomorphic functions of complex analysis. Famously, harmonic functions do not have singularities. A quantitative version of this fact is that a solution of the heat equation is spatially two derivatives more smooth than the steady external heat applied to the equation. We label this as a gain of two derivatives. A celebrated "Bracket condition" of Hörmander links a wide class of degenerate laplacians with a quantitative gain of fewer than two derivatives and excludes singularities. Fractional derivatives, which I will define in the talk, quickly come in handy. What happens in the limit when even fractional derivatives are not enough to describe quantitative smoothness gain? Are singularities possible?

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