

BI-CO MATHEMATICS COLLOQUIUM

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*“Submajorization and the Geometry of Unordered
Collections, with Applications to Music and
Welfare Economics”*

Monday, October 6, 2014

Talk at 4:00 – Park 338
Tea at 3:30 – Park 355, Math Lounge

Abstract:

If several voices or musical instruments sound a sequence of chords, our ears track the movement in each voice. This association is called voice leading. Although composers are normally instructed to “minimize” the overall amount of vocal movement, there is no principled way of choosing a measure for vocal movement when multiple individuals are involved. A similar problem arises in welfare economics: if we wish to measure income volatility, we must measure multiple income changes simultaneously. Tymoczko (2006) proposed requirements that every method of comparing voice leadings should satisfy. These requirements are equivalent to the submajorization partial ordering, defined by Hardy, Littlewood, and Polya (1934) and developed in the study of welfare economics. Moreover, submajorization may be used to compare distances in the orbifolds (quotients of \mathbb{R}^n) representing the geometrical space of “chord types” and to measure distances between melodies of equal length. (This represents joint work with Dmitri Tymoczko, Princeton University.)

A musical orbifold: The set of two- and one-note chords forms a thrice-twisted Möbius strip, with the one-note chords on the boundary forming a trefoil knot.

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