

A Tale of Two Series

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Infinite series received their first serious treatment when Jakob Bernoulli published his *Tractatus de seriebus infinitis* in 1689. Ever since, this subject has been a staple of calculus courses, here at Bryn Mawr and beyond.

But if you think you know all there is about infinite series, you might be in for a surprise. Leonhard Euler, the greatest mathematician of the 18th century, found ways to sum the seemingly unsummable. In this regard, I have a pair of examples in which his genius is on full display.

The first is the “Basel problem,” where Euler determined the *exact* value of

$$1 + 1/4 + 1/9 + 1/16 + 1/25 + \dots + 1/n^2 + \dots$$

I’ll present not his original derivation from 1734 but a lesser-known argument from 1755 in which he (improbably) used l’Hospital’s rule.

Building on this, Euler evaluated a second infinite series so bizarre that finding its exact sum seemed hopeless. But he did it. As my second example, this stands as a jaw-dropping demonstration of mathematical talent.

If you want to look over the shoulder of one of history’s greatest mathematicians, please stop by the Distressing Math Collective (DMC) on **Thursday October 12** at **7 p.m.** in **Park 338**.

NOTE: The talk will be accessible to anyone who has completed the calculus sequence.