

# *The Empty Place in My Space: How $\mathbb{Q}$ Complete Me*

**Daniel White**

We all know one metric on the rationals: the one induced by absolute value. Did you know there are others? For each prime  $p$  there is an associated metric on  $\mathbb{Q}$ , induced by what's known as the  $p$ -adic norm. These appear naturally when one attempts to classify the norms that can be placed on  $\mathbb{Q}$  and are the only others that exist up to some equivalence. The rabbit hole doesn't end there! Remember that  $\mathbb{R}$  is the completion of  $\mathbb{Q}$ , i.e. what appears when you "fill in the holes". There's nothing stopping us from completing  $\mathbb{Q}$  when equipped with these new fancy metrics, and we obtain fields known, each known as the  $p$ -adic numbers.

At this talk, we will discover how the  $p$ -adic norms appear as we classify all norms on  $\mathbb{Q}$ , a result due to Ostrowski. We'll then discuss one method of constructing  $\mathbb{R}$  from  $\mathbb{Q}$  with the absolute value and use that same construction to produce the  $p$ -adic numbers.

**Date: Wednesday February 13, 2019**

**Time: 7:00 pm**

**Place: Park 328**