Math 215: Linear Algebra, Spring 2020

Instructors: Tarik Aougab (he/him/his) and Chung-Nan Tzou (he/him/his). Discussion sessions led by Jeff Tecosky-Feldman (he/him/his)

Office Hours: Will be announced during the second week of class after we get a feel for everyone's schedules and can pick times that work for all. If you'd like to meet during the first week of class, please feel free to set up an appointment via email with either of us.

Textbook: Linear Algebra and its Applications, by Lay, Lay, and MacDonald. If obtaining a copy is difficult for any reason (e.g. it's too expensive), we can make a copy available and we can certainly make the homework problems available. Please email either Tarik or Chung-Nan with any questions or concerns about purchasing textbooks.

Course Description and objectives: Linear algebra is absolutely essential in mathematics, engineering, physics, statistics, and most other mathematically related sciences. It is a bridge between algebra and geometry: on the one hand it allows you to describe lines, planes, and other geometric objects in terms of algebraic equations; on the other, given a set of equations, we can use linear algebra to trade in those equations for a collection of lines, planes, etc. and then use the geometry of those objects to reason about the original equations. It's an extraordinarily powerful tool and in this class we will learn how best to use it. Even without all of its usefulness, linear algebra is also fascinating because it allows us to understand, concretely, the concept of "dimension". What does it *really* mean to say that we live in three dimensions? Can we make any sense at all of a 17-dimensional space? By the end of the semester, we'll be able to tackle these questions.

Goals for the course: The course will be a mixture of *theory* and *practice*. One of our goals will be to become familiar with how to use linear algebra to solve complicated systems of equations and to become familiar with the concepts of an abstract vector space, linear transformations, eigenvalues, eigenvectors, and quadratic forms (*we do not expect any of you to have ever heard of these terms before! We'll learn what they mean, and more, during the semester*).

In addition to learning how to use linear algebra in practice, in Math 215 students will also practice the skill of abstract mathematical reasoning and the art of mathematical proof. As opposed to just being told that a particular fact about linear algebra happens to be true, we will dive much deeper and mathematically justify that it is true, beyond a shadow of a doubt.

Coming up with and writing proofs can be extremely difficult! We do not expect students in Math 215 to have any exposure to mathematical proof at the beginning of the semester, so please do not worry if you have no experience with proofs. A major focus of the course will be to introduce students to proofs in lecture and to guide them towards creating their own proofs on assignments.

Why proofs? What is the point of proof and why do we emphasize them in mathematics?

When you've proven something, you not only know it's true, but additionally you understand *how* or *why* it's true. So if you're interested in deep and conceptual understanding of the tools you're using (as opposed to just being handed a formula, being told to trust that it works and to not question where it came from), proofs become very useful.

We would also like to emphasize that *even if you're only interested in applying the tools that we learn*, it will not be enough to just learn a few "recipes" and to become proficient with them. This is because when dealing with hard problems in the real world, it will almost never be the case that some formula or recipe you learned in your linear algebra class will apply directly to your situation. You'll have to tweak, alter, or even completely replace those recipes because the real world is complicated! And to do this, you really need to know why the recipe works and what the logic is behind it. If you treat a formula like magic, you won't be able to dissect it later on and alter it when you need something a bit different!

Our course will sometimes feel like we're doing two very different things at once: (1) practicing with the recipes of linear algebra and doing computations, and (2) justifying those recipes through proof and studying abstract mathematical concepts. Try to think of these two halves as being deeply related to one another; in particular, learning the theory behind the recipe is essential to honing our practice and our ability to apply linear algebra in the real world.

Growth, not ability: There is a very prevalent belief that you are either "good" or "bad" at math, and if you are "bad" at it, then you will always be bad at it no matter how hard you try. **This is extremely false, and the mathematics community bears a lot of responsibility for perpetuating this myth**. In reality, mathematics is just like any other discipline or skill: you can improve more and more with practice.

We are all capable of growth in mathematics. You should measure your success in this class by how much your understanding of the concepts have improved over the course of the semester. Also, like most things worth doing, math is hard, so you should expect to struggle with the material! When you struggle, you are learning and growing. If you find that you are not struggling at all, this might not be the right course for you and you should consider a more advanced course.

Prerequisites: Solid understanding of calculus and pre-calculus.

Course structure:

Class time: class time will be held in an interactive lecture format with space for questions and conversation. It can be extremely helpful to try to read over the relevant sections in the textbook *before* we cover them in class. If you do this, you should not expect to understand everything, or even most of what you see! Don't worry about this and don't take this exercise too seriously; try to get an extremely basic feel for whatever the topic is and if it looks complicated or confusing, try also to formulate some questions to bring to class.

Discussions: Weekly discussions in smaller groups, hosted by Jeff, are scheduled for Tuesdays. These discussions are a very important part of the course because they give you an opportunity to discuss the week's main ideas in a smaller group setting and to get some additional practice with an instructor on stand-by.

Homework: Mathematical proficiency requires practice, and a lot of it. The concepts we are going to learn aren't easy to grasp and we expect that they will require a good deal of contemplation outside of class to really understand. So, there will be a lot of homework. **Students should expect to spend at least 6-8 hours a week on outside homework assignments**.

Keep in mind that the point of mathematical proof is not only to convince yourself that something is true, *but also to communicate that truth effectively to one another!* This can not be stressed enough. Mathematics is a social activity, practiced by human beings working together to understand our world and our universe. That means that an important component of proof-writing is learning how to write in a way that is clear, concise, and inviting! Just like writing an essay in a history, English, or philosophy course, there is a certain "style" that goes along with writing good mathematical proofs.

The homework grading scheme reflects the fact that this is in some sense a writing course in as much as it is a course in abstract mathematical concepts. Each homework problem will be graded out of 4 points. Here is a basic rubric:

4 points: A complete and correct solution with no logical gaps, and clear/concise exposition.

3 points: A complete and correct solution with no logical gaps, but the exposition needs work OR a good attempt with clear exposition that would be fully logically correct if some minor details were fixed.

2 points: A good attempt with clear exposition but with a significant logical gap or error in the argument OR a good attempt with some minor logical issues AND the exposition needs work.

1 point: The beginnings of some ideas are present.

0 points: No serious attempt at a solution has been made.

Course policies and recommendations for homework:

We strongly encourage students to work together on the homework assignments. Collaboration is an incredibly important aspect of mathematical science and so you should think of the homework assignments as an opportunity to practice the skill of working well with others. On the other hand, it is very easy to "trick" yourself into thinking you understand something when working with a group of peers who come to an answer collectively. So, be sure that you are writing up your own solutions and that you understand the ins and outs of each problem. To do this, you should first think about the problems on your own, get a grasp of what they're asking and how they might be approached, and then meet up with your collaborators to work together from there. **If you do work in a group for the homeworks, you must turn in your own assignment, written in your own words. You must also mention who you worked with on your assignment so that the graders can keep track of who worked with whom.**

Respecting each other: We are not all coming to this class with the same privileges, resources, time, and knowledge. It's really important to keep this in mind when working with each other on homework assignments and during lecture. **It is our strong belief that as a community, mathematicians and scientists need to do a much better job of making our disciplines more accessible to people of all races, genders (including gender non-conforming folks), sexual identities, and class backgrounds**. While this is a priority for us in the classroom, we do not claim to know how to best honor this commitment, and so **we are very open to feedback from students when it comes to making the course more accessible and inclusive to all identities**.

It's also important to think about how to respect one another when working together on homework assignments. It's not equally easy for all of us to speak up in a large group, and the voices of historically underrepresented/marginalized students are most easily drowned out in group work. So please keep this in mind when working together. Here are some concrete examples of positive collaborative behavior:

(1) Making sure everyone who wants it has the opportunity to speak frequently. This can mean checking in with each other to make sure everyone is following along and contributing when they have an idea.
(2) Respecting people's pronouns and other aspects of their identity.
(3) Making sure that everyone's ideas are acknowledged when writing up the final solution to a problem. When working in groups, solutions often evolve organically; an idea might pop into your head and you may think it's yours and yours alone, but perhaps you only arrived there because of something else that someone already said. Pay attention to what people are saying and try to learn from one another.
(4) Honoring different types of contributions. In group work, *active listening* can be just as valuable as speaking. If you have ever processed a thought or an idea by talking it out with an attentive friend, you've experienced this truth firsthand. Listening carefully is in and of itself an important and difficult skill. It can involve being silent and creating the space for your partner(s) to say what's on their mind, and it can also involve asking the right insightful question at the right time. Practice listening with your collaborators, and also practice recognizing and appreciating when your collaborators are actively listening to you!

We will do our best to check in with folks periodically during the semester. **If at any time in the semester you want to be working in a group but do not have a group of students to work with, please let us know and we will help you find a working group. If at any time in the semester, you find yourself in a group of students for which the above behaviors aren't being practiced and people aren't feeling respected, please let us know as well.**

Grading and exams:

There will be two midterm exams which will both have a closed-book in-class component and a short open-book take-home component. The purpose of having two components is because we know that in-class, timed exams can be very stressful for students. So, we want to take some of the timed pressure off of you by giving you the opportunity to show what you've learned in the take-home component.

(1) Webwork assignments: 10 %
(2) Written Homework: 15 %
(3) Midterm 1: 20 %
(4) Midterm 2: 20 %
(5) Final Project: 10 % (OPTIONAL)
(6) Final exam: 25 % or 35 % depending on final project

We do not like any one test to have to count for more than a quarter of the final grade. So in order to take some of the pressure off of the final exam, we are offering students the opportunity to submit a **final project**. The project will be due towards the end of the semester, and it will consist of a detailed write-up/summary of a real-world application of linear algebra. For any student who completes the final project, the final exam will be worth 25 % of the grade. For a student who does not complete the final project, the final exam will be worth 35 % of the grade.

Here are some possible examples:

(1) Google– it turns out that the key idea behind most internet search engines is linear algebra.

(2) Markov processes– using linear algebra to understand the "long-term" behavior of a statistical process.
(3) Network analysis– using linear algebra to study large networks of people, economies, ideas, etc. etc. In fact example (1) is sort of a special case of this one.

The final project should:

(1) be at least 5 pages long (standard font size and margins, single-spaced,);
(2) include figures that can either be computer generated or hand-drawn, depending on the student's preference;
(3) Not be plagiarized! Please be sure to cite all sources used and to put everything in your own words;
(4) include a model example of the application, in which the student cooks up an *original* hypothetical situation with specific numbers, and then uses the application to solve some problem related to the example. e.g., when Google is using linear algebra, it's doing it in order to analyze millions of websites. A good example of what we are looking for would be coming up with a "model" of the internet with maybe only ten websites, and then applying the key idea to those ten websites.

Resources: Haverford College is committed to supporting the learning process for all students. Please contact one or both of us as soon as possible if you are having difficulties in the course. There are also many resources on campus available to you as a student, including the Office of Academic Resources (https://www.haverford.edu/oar/) and the Office of Access and Disability Services (https://www.haverford.edu/access-and-disability-services/). If you think you may need accommodations because of a disability, you should contact Access and Disability Services (athc-ads@haverford.edu). If you have already been approved to receive academic accommodations and would like to request accommodations in this course because of a disability, please meet with one of us privately at the beginning of the semester (ideally within the first two weeks) with your verification letter.