

Philadelphia Area Number Theory Seminar

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Apollonian Packings and Kac-Moody Root Systems

Abstract:

Fix four mutually tangent circles in the plane. Fill in the spaces between these circles with additional tangent circles. By repeating this process ad infinitum, on smaller and smaller scales, we obtain an Apollonian circle packing. I will define a four-variable generating function for curvatures that appear in an Apollonian packing. This function is essentially a character for a rank 4 indefinite Kac-Moody root system. I will relate this generating function to certain automorphic forms, including theta functions on $SL(2)$ and a Siegel automorphic form on $Sp(4)$. And I will discuss its domain of convergence, the Tits cone of the root system, which inherits the rich geometry of Apollonian packings. Finally, I will discuss generalizations to other packings and root systems.

Thursday, September 23, 2021
3:25 – 4:45 PM

Bryn Mawr College
Department of Mathematics
Park Science Center **245**

Informal refreshments at 3:10PM in Park 361