

# Philadelphia Area Number Theory Seminar

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## Extreme values of twisted $L$ -functions

**Abstract:** Distribution of values of  $L$ -functions on the critical line, or more generally central values in families of  $L$ -functions, has striking arithmetic implications.

One aspect of this problem are upper bounds and the rate of extremal growth. The Lindelöf Hypothesis states that  $\zeta(\frac{1}{2} + it) \ll_{\epsilon} (1 + |t|)^{\epsilon}$  for every  $\epsilon > 0$ ; however neither this statement nor the celebrated Riemann Hypothesis (which implies it) by themselves do not provide even a conjecture for the precise extremal sub-power rate of growth. Soundararajan's method of resonators and its recent improvement due to Bondarenko–Seip are flexible first moment methods that unconditionally show that  $\zeta(\frac{1}{2} + it)$ , or central values of other degree one  $L$ -functions, achieve very large values.

In this talk, we address large central values  $L(\frac{1}{2}, f \otimes \chi)$  of a fixed  $GL(2)$   $L$ -function twisted by Dirichlet characters  $\chi$  to a large prime modulus  $q$ . We show that many of these twisted  $L$ -functions achieve very high central values, not only in modulus but in arbitrary angular sectors modulo  $\pi\mathbb{Z}$ , and that in fact given any two modular forms  $f$  and  $g$ , the product  $L(\frac{1}{2}, f \otimes \chi)L(\frac{1}{2}, g \otimes \chi)$  achieves very high values. To obtain these results, we develop a flexible, ready-to-use variant of Soundararajan's method that uses only a limited amount of information about the arithmetic coefficients in the family. In turn, these conditions involve small moments of various combinations of Hecke eigenvalues over primes, for which we develop the corresponding Prime Number Theorems using functorial lifts of  $GL(2)$  forms.

This is part of joint work on moments of twisted  $L$ -functions with Blomer, Fouvry, Kowalski, Michel, and Sawin.

Wednesday, October 10, 2018, 2:40 – 4:00 PM

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Park Science Center **328** · Tea and refreshments at 2:20PM in Park 361