Bryn Mawr College
Department of Physics
Mathematics Readiness Examination
for Introductory Physics

Answers

1. CHOICE D: We are given $x - 1 = 2$. To solve for x, add 1 to both sides of the equation:

   $x – 1 = 2$
   $+ 1 = + 1$

   $x = 3$

   so $x + 1 = (3 + 1) = 4$

2. CHOICE B: volume = $\pi R^2 h = (3)(2 \text{ cm})^2 (5 \text{ cm}) = 60 \text{ cm}^3$

3. CHOICE C: If $x = 3$ then $x^2 + 3 = 3^2 + 3 = 9 + 3 = 12$

4. CHOICE C: The area is 8 entire squares plus 0.8 + 0.4 + 0.9 + 0.1 + 0.5 squares which is 10.7 squares. Each square has an area of so the total area is about 53.5.

5. CHOICE A: $\frac{(-2)(-6)}{-4} = \frac{12}{-4} = -3$

6. CHOICE D: $(2xy^3)^3 = 2^3x^3(y^3)^3 = 8x^3y^9$

7. CHOICE A: $(2x – 1)(4x + 1) = 2x(4x + 1) + (-1)(4x + 1)$

   $= 8x^2 + 2x – 4x – 1$

   $= 8x^2 - 2x – 1$

8. CHOICE A: $\frac{4\times10^{-15}}{8\times10^{-12}} = 0.5\times10^{-15+12} = 0.5\times10^{-3} = 5\times10^{-4}$.

9. CHOICE D: A common demominator is necessary:

$$\frac{x^2}{y} + \frac{x}{y^2} \quad \text{multiply the first term by } \frac{y}{y} \text{ to get}$$

$$\frac{x^2 y}{y^2} + \frac{x}{y^2} = \frac{x^2 y + x}{y^2}$$

10. CHOICE C: This is the difference between two squares:

$x^2 - 100 = (x - 10)(x + 10)$

11. CHOICE A: $\left(5 \times 10^8\right)\left(6 \times 10^{-12}\right) = 30 \times 10^{8-12} = 30 \times 10^{-4} = 3 \times 10^{-3}$

12. CHOICE A: $(2x + 3) - (x - 2) = 2x + 3 - x + 2 = x + 5$

13. CHOICE C: $A^2 + B^2 = C^2 = 1 + 3 = 4$. So $x = 2$.

14. CHOICE C: Let x be the number. “Of” means multiply, “is” means equals:

$$\frac{1}{3}(x) = 8$$

Multiply both sides by three: $x = 24$

One-fourth of 24 is:

$$\frac{1}{4}(24) = 6$$

15. CHOICE A: $x^3 y = (-2)^3 5 = (-8)(5) = -40$

16. CHOICE B: 25 m = (25 m) (3 feet/m) = 75 feet.

17. CHOICE C: $(x^2 - 3x + 2) - (3x^2 - 5x - 1)$

$= x^2 - 3x + 2 - 3x^2 + 5x + 1$

$= -2x^2 + 2x + 3$

18. CHOICE D: $\frac{2x}{3y} \cdot \frac{9y}{4x^2} = \frac{2\cancel{x}}{3\cancel{y}} \cdot \frac{9\cancel{y}}{4x^{\cancel{2}}} = \frac{3}{2x}$

19. CHOICE C: Factor the polynomial: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

20. CHOICE D: $ln(ab) = ln(a) + ln(b)$

21. CHOICE C: We are asked for the absolute value: $|3 - 8| = |-5| = 5$

22. CHOICE A: We need a common denominator, and xy is a good choice:

$$\frac{2}{x} = \frac{2y}{xy} \quad \text{and} \quad \frac{5}{y} = \frac{5x}{xy}$$

Adding the new expressions gives:

$$\frac{2y}{xy} + \frac{5x}{xy} = \frac{2y + 5x}{xy}$$

23. CHOICE E: Top and bottom are square, so each has an area of x^2. Each side (four of them) has area xh, so total surface area is $2x^2 + 4xh$.

24. CHOICE C: $x - y = (-4) - (-7) = -4 + 7 = 3$

25. CHOICE D: $f(x) < 0$ whenever the graph is below the x-axis: $x < -1$ or $x > 3$

26. CHOICE D: In 20 years, there are four doubling periods (each 5 years), so the money increases by a factor of $2 \times 2 \times 2 \times 2 = 2^4 = 16$

27. CHOICE B: The second graph is the only one that is symmretical with respect to the y-axis, and thus even.

28. CHOICE D: Subtract $3y$ from and add 4 to both sides of the equation:

$$7y - 4 = 16 + 3y$$
$$-3y + 4 \quad -3y + 4$$
$$4y = 20$$

Divide both sides by 4:

$$\frac{4y}{4} = \frac{20}{4} = 5$$

29. CHOICE D: $\frac{10(-2)(-3)(-1)}{5} = \frac{60}{5} = 12$

30. CHOICE A: The graphs of $x - 2y = 6$ and $x + y = -3$ intersect at the values of x and y that satisfy both equations. To get these, solve the two equations simultaneously by solving the first equation for $x = 2y + 6$. Substitute into the second equation:

$(2y + 6) + y = -3$

$= 3y + 6 = -3$

Subtract 6 from both sides:

$$\begin{array}{l} 3y + 6 = -3 \\ \quad\;\; -6 \quad -6 \\ \hline 3y = -9 \end{array}$$

Divide both sides by three:

$$\frac{3y}{3} = \frac{-9}{3}$$

$y = -3$

31. CHOICE E: $8^{-1/3}9^{1/2} = \frac{1}{2} \times 3 = \frac{3}{2}$

32. CHOICE B: $\sqrt[3]{-27} = -3$ because $(-3)(-3)(-3) = -27$. Remember that third roots can be negative!

33. CHOICE A: As x becomes very large and positive, y becomes very large because the term in x^2 increases much faster than that in x. The same is true as x becomes very negative. Also recall an equation of the form $ax^2 + bx + c$ is a parabola.

34. CHOICE D: Recall that $\log_a(b) = c$ means $a^c = b$.

$\log_3(x + 1) = 2$ means

$3^2 = x + 1$

$9 = x + 1$ Subtract one from both sides:
-1 -1

$x = 8$

35. CHOICE D: $(-2x^2)(3x^2y)(-y) = 6x^4y^2$

36. CHOICE C: As x becomes very negative, 3^x becomes very small (i.e. $3^0 = 1$) and as x becomes large and positive, 3^x becomes very large.

37. CHOICE B: Since the expression is equal to zero, we can ignore the value of the denominator and set the numerator equal to zero. Thus,

$(2x + 1)(x - 1) = 0$

This expression holds when either factor is zero:

$2x + 1 = 0$
- 1 -1

$2x = -1$
--- ----
2 2

$x = 1$

$x - 1 = 0$
+1 +1

$x = 1$

Thus, $x = -½, 1$

38. CHOICE B: $13a - 15b - a + 2b$ Factor with respect to a and b:

$= (13 - 1)a + (-15 + 2)b = 12a - 13b$

39. CHOICE D: $3^{14} = (3^7)^2 \approx (2000)^2 \approx 4\times10^6$.

40. CHOICE B: The length of segment BC is 6, while the length of segment AB is 8. Since we have a right triangle, we can use Pythagorean Theorem:

$a^2 + b^2 = c^2$

Let c be the hypotenuse, or the unknown. Then,

$6^{2+}8^2 = c^2$

$36 + 64 = 100 = c^2$, so $c = 10$.

41. CHOICE C: Substitute $a + 2$ in for x:

$$f(a+2) = \frac{2(a+2)+6}{(a+2)+2} = \frac{2a+10}{a+4}$$

42. CHOICE D: We know the graph is a line because x appears only to the first power, and falling to the right because its slope (the coefficient of x), is negative.

43. CHOICE B: Subtract b from both sides:

$$\begin{array}{l} ax + b = 3,\ a \neq 0 \\ \quad -b \quad -b \\ \hline ax = 3 - b \end{array}$$

Divide both sides by a:

$$\frac{ax}{a} = \frac{3-b}{a}$$

$$x = \frac{3-b}{a}$$

44. CHOICE C: $a + b$ is a factor of $a^2 - b^2 = (a + b)(a - b)$ and of $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

45. CHOICE D: Subtract p from both sides of the equation:

$$\begin{array}{l} 3p > p + 12 \\ -p \quad -p \\ \hline 2p > 12 \end{array}$$

Divide both sides of the equation by 2:

$$\frac{2p}{2} > \frac{12}{2}$$

$p > 6$

46. CHOICE A: tangent is opposite over adjacent.

47. CHOICE D: $A^{ab} = \left(A^a\right)^b = \left(A^b\right)^a$

48. CHOICE B: The height of the rectangle occurs where the curve intersects the rectangle, at $x = 0.5$. We can find the value of y at $x = 0.5$ by substituting 0.5 for x:

$(0.5)^2 + 3(0.5) - 1 = 0.75$

The area of the rectangle is thus (0.75) (.2) = 0.15.

49. CHOICE D: $xy \rightarrow (2x)(2y) = 4xy$

50. CHOICE D: any finite quantity (including zero) raised to the zeroth power = 1.

51. CHOICE C: $4 - (-2 + 5) = 4 - (3) = 1$

52. CHOICE E: sine is opposite over hypotenuse = 3/D. Using Pythagorus' Theorem, D = 5. So sin(b) = 0.3/0.5 = 0.6.

53. CHOICE D: $|x - 2| \leq 1$ is equivalent to $1 \leq x \leq 3$.

If $x > 2$, then $(x - 2)$ is positive and $|x - 2| = x - 2 \leq 1$, which means $x \leq 3$.

If $x < 2$, then $(x - 2)$ is negative and $|x - 2| = -(x - 2) = 2$, or $-x + 2 = 2$, so $x \geq 1$.

54. CHOICE C: $\dfrac{3/2}{2/3} = \dfrac{3}{2}\dfrac{3}{2} = \dfrac{9}{4}$

55. CHOICE C: Let l be the length of the rectangle, and w its width.

$l = 2w + 3$

We are given the perimeter, $2l + 2w = 90$.

Using the first equation in the second:

$2(2w + 3) + 2w$

$= 6w + 6 = 90$
$\quad - 6 \quad - 6$

$6w = 84$
--- ---
6 6

$w = 14$

56. CHOICE A: $4(s + 2) = (4 \times s) + (4 \times 2) = 4s + 8$

57. CHOICE A: $3/4 - 1/7 = \frac{3}{4} - \frac{1}{7} = \frac{21-4}{28} = \frac{17}{28}$

58. CHOICE B: Subtract one from both sides:

$1 - 5x < 3$
$- 1 \quad - 1$

$- 5x < 2$

Divide both sides by -5, and remember to switch the sign of the inequality because we are dividing by a negative number:

$- 5x < 2$
----- ---
-5 -5

$x > - 2 / 5$

59. CHOICE B: The function has an absolute minimum at $x = 1$, the lowest point on the graph between 0 and 4. The other low point at $x = 3$ is a "local minimum."

60. CHOICE A: $3^2 + 4^2 = D^2 = 25$ so $D = 5$.

61. CHOICE B: $\left(2\sqrt{3}\right)\left(3\sqrt{6}\right) = 6\sqrt{18} = 6\sqrt{(2)(9)} =$
$6\sqrt{9}\sqrt{2} = (6)(3)\sqrt{2} = 18\sqrt{2}$

62. CHOICE B: $1 - \sin^2\theta = \cos^2\theta$ (a trigonometric identity).

63. CHOICE A: $f(x) = \cos(3x)$, then $f(\pi / 6) = \cos(\pi / 2) = 0$.

64. CHOICE A: The circumference of a circle is $2\pi R$.

65. CHOICE E: The sine curve has a *y*-intercept at zero, increases as *x* increases to π/2 and decreases as *x* decreases to - π/2.

66. CHOICE E: cscθ = 1/sinθ and tan θ = sin θ/cos θ, so
$\sin\theta \tan\theta \csc^2\theta = \sin\theta (\sin\theta/\cos\theta)(1/\sin^2\theta) = 1/\cos\theta = \sec\theta$.

67. CHOICE B: tan θ = sin θ/cos θ, and cos (- π/2) is zero. A zero in the denominator renders the expression undefined.

68. CHOICE E: The area of a circle is πR^2

69. CHOICE B: the sum of the angles in a triangle add up to 180 degrees.

70. CHOICE C: Taking the slope between *x* = 0 and *x* = 5, we see that:

$$\text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{20-5}{5-0} = \frac{15}{5} = 3$$

71. CHOICE E:

$$\left(\frac{100\ km}{minute}\right) = \left(\frac{100\ km}{minute}\right)\left(\frac{5\ miles}{8\ km}\right)\left(\frac{1\ minute}{60\ seconds}\right)$$

$$= \frac{500\ miles}{480\ seconds} = 1\,\frac{mile}{second}$$